Determination of optimal frame sizes in framed slotted ALOHA

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The primary task of radio-frequency identification (RFID) readers is to identify multiple objects as quickly and reliably as possible with minimal power consumption and computation. An optimisation problem related to the selection of appropriate frame sizes for the framed slotted ALOHA protocol, commonly used in passive RFID tag identification systems, is considered. The fundamental question concerning this issue is: what frame size is most suitable for identifying as many tags as possible in a given time? More formally, what is the optimal value for frame sizes in the sense that the probability of successful identification is maximised? Thus far, many research results related to this problem have been reported in the literature, but most are either suboptimal or empirical. Through asymptotic analysis, it is shown that the optimal frame size is approximately given by $N/\ln 2$ for $N$ tags.

Introduction: The radio-frequency identification (RFID) market was worth $8 billion in 2013, up from $7 billion in 2012, and is currently growing to $9 billion in 2014. The IDTechEx forecast is that it will rise to $30 billion in 2024 [1]. Rapid growth is being witnessed, in particular in applications such as apparel tagging in the retail sector. RFID-based ticketing in public transport systems and the tagging of animals (such as pets, sheep and cattle). It is noteworthy that the source of most of this growth is passive ultra-high-frequency RFID tags, mainly because of their low cost and the ease of large-scale deployment [1, 2].

RFID systems usually consist of readers (also called interrogators) and tags (or transponders). A typical system has a few readers, either stationary or mobile, and many tags, which are attached to objects, such as pallets, cartons and bottles. A reader communicates with the tags in its wireless range and retrieves identification (ID) codes emitted by tags. Among three types of commercially available tags—passive, semi-passive and active—passive tags are the least complex and hence the cheapest. They have no internal power source and, therefore, use the electromagnetic field transmitted by a reader to power their internal circuits.

Collisions resulting from simultaneous tag responses are one of the key issues in RFID systems. Collisions waste bandwidth and energy and increase tag ID delays. Hence, algorithms need to be devised to minimise collisions between the tags and reader. The task of designing anti-collision protocols is rendered more challenging by the fact that the tags are required to be simple, cheap and sufficiently small [3]. Among several anti-collision protocols proposed thus far, the framed slotted ALOHA (FSA) protocol is the most widely used, owing to its performance and simple implementation [4]. In this Letter, we consider some optimisation problems (OPs) associated with FSA frame sizes.

Consider a passive RFID tag ID process with $N$ passive tags located in the reader’s interrogation zone. The primary task of the reader is to identify all $N$ tags in the shortest possible time. The overall tag-reading procedure can be outlined as follows [3, 5]. At the beginning of each FSA frame, the frame size, i.e. the number of time slots, $L$ for the upcoming FSA frame is broadcast to the tags, and during the reading frame, every tag transmits its ID in a randomly chosen time slot. Meanwhile, the reader retrieves the IDs successfully transmitted by tags while collecting statistics on the number of slots filled with zero, one or multiple tag responses (a slot with multiple tag responses implies a collision within the slot). The tag-reading procedure is repeated frame-to-frame in the same manner until the reader terminates the process.

One issue addressed in this Letter is the most efficient frame sizes; that is, what is the suitable value for frame sizes such that as many tags as possible are identified in a given time? More formally, with a fixed total ID time $K$ (in time slots), what is the optimal value of $L$, $L_{\text{opt}}$, in the sense that the probability that a tag makes at least one successful transmission through $K$ slots (referred to as the ‘probability of successful ID’ hereafter) is maximised?

In reality, the number of tags $N$ is unknown to the reader and the reader should estimate it using an appropriate tag estimation function, utilising the (approximated) collected statistics at the end of each frame. The tag estimate, namely, $\tilde{N}$, is again utilised in computing the appropriate frame sizes to be used for the following frames. In this scenario, frame size $L$ is adjusted according to $\tilde{N}$ at appropriate time instances (e.g. on a frame-by-frame basis) during the ID process. Consequently, the so-called dynamic FSA protocol is more realistic than FSA. However, in this Letter, we assume that the frame size is fixed throughout the ID process in order to facilitate discussion, since our interest is focused mainly on the optimal selection of frame sizes for a given $N$ (i.e. we consider the situation after $N$ or $\tilde{N}$ is determined).

In addition, we assume that each tag transmits its ID once every frame, regardless of whether the previous transmissions were successful; i.e. the muting function is not available. Therefore, it is noteworthy that the results herein are not restricted to the FSA protocol.

Consider the case $(N, K) = (10, 32)$. In general, for given $(N, K)$, any integers from 2 to $K$ can be used; however, we restrict this to the integers by which $K$ is divisible for simplicity. Thus, the possible values of $L$ form a set $\mathcal{A}_K = \{2, 4, 8, 16, 32\}$. Suppose that we select two external values 2 and 32. Frame size $L=32$ implies that a single reading frame with 32 time slots is utilised, whereas $L=2$ implies 16 reading frames with two time slots. In other words, for $L=32$, the contention for time slots among all tags is the least competitive, but only one chance for ID transmission is available to each tag. However, for $L=2$, the contention for time slots among tags is the most competitive, but each tag can transmit several, i.e. 16 times. Then, we can raise the following questions: Which value is better in terms of the probability of successful ID? What is the optimal value for $L$?

In this Letter, we show that, for given $(N, K)$, $L_{\text{opt}} = \arg \min_{L \in \mathcal{A}_K} (L - N/\ln 2)$, i.e. the integer in $\mathcal{A}_K$ that is the closest to $N/\ln 2$. In the following Section, we formulate the OP, and then proceed to the solution as well as the introduction of related research results.

Optimisation problems: In this Section, we formalise the OP related to the selection of appropriate FSA frame sizes. First, it should be noted that we consider only the cases where the tags are not informed by the reader about the outcome of each reading frame. Therefore, in the case of multiple reading frames, each tag transmits its ID once every frame regardless of whether or not the previous transmissions were successful.

Suppose $N$ tags are present in the interrogation zone. Focusing on a specific tag, we seek the value of $L$ that maximises the probability that the tag makes at least one successful transmission within $\gamma$ frames, i.e. $P_{\text{opt}}^\gamma$, termed the probability of successful ID.

Optimisation problem: With $K$ fixed, determine the frame size $L_{\text{opt}}$ such that $P_{\text{opt}}^\gamma$ can be maximised, i.e.

$$L_{\text{opt}} = \arg \max_{L \in \mathcal{A}_K} P_{\text{opt}}^\gamma$$

(1)

Preliminaries: In this Section, we summarise our literature survey results concerning the OP established in the preceding Section. For the slotted ALOHA with $N$ tags and frame size $L$, the average throughput $U$, i.e. expected number of tags successfully identified during an FSA frame, is given by

$$U = N \left(1 - \frac{1}{L}\right)^{N-1}$$

and the normalised throughput $U_{\text{norm}}$ (often referred to as system efficiency in the literature) is given by

$$U_{\text{norm}} = \frac{N}{\left(1 - \frac{1}{L}\right)^{N-1}}$$

Normalised throughput $U_{\text{norm}}$ represents the utilisation of FSA slots in the ID process and can be interpreted as the probability that a slot will have a single transmission during an FSA frame. Normalised throughput is maximised at $L=N$ with a maximum of approximately $e^{-1}$ for large $N$.

Floerkemeier et al. [6] and Bueno-Delgado and Vales Alonso [9] proposed using $L=N$ as the optimal value from the perspective of maximising normalised throughput. Similarly, Huang [10] suggested the formula $L_{\text{opt}} = 1/(1 - e^{\ln H/2/N})$. He should be pointed out that all of these results have one commonality in that they are obtained in terms of normalised throughput or system efficiency and are not optimal with respect to the OP addressed in this Letter. On the other hand, Khandelwal et al. [11] proposed using $L_{\text{opt}} = 1.943N$, where $H = \ln(2N)/(\ln(1-1/N))$, with $H$ representing the number of slots with no tag response.
Similar results also can be found in the literature: Zhen et al. [3] proposed $L_{opt} = 1.4 \times N$, based on their experimentations (it should be noted that $1/\ln 2 \approx 1.4427$ in our result $L_{opt} = N/\ln 2$). Vogt [5] provided a frame size table that listed some of the recommended values for several ranges of $N$. Prodanoff [12] also considered the asymptotic regime (i.e. the case where $N$ is very large) and derived the formula $N/\ln 2$, the same as ours. However, the nature of the results presented in [12] is quite different from that of ours basically on two points. First, our results are more generalised in that they are derived under much fewer constraints and answer the fundamental question: What frame size is most suitable for maximising the probability of successful ID? The problem setting in [12] was rather specific: the results were obtained in terms of 'census delay' (similar to but different from ID delay), defined as the minimum number of time slots required for the probability of individual successful transmission to exceed a given confidence level. Secondly, our Letter provides a mathematically well-established OP concerning the performance of the FSA protocol, and the proof is relatively short and concise, while the proof given in [12] is excessively long and some definitions therein (e.g. census delay and confidence level) are somewhat ambiguous.

Proposed approach to solution of OP: Let us return to the OP. The probability of successful ID $P_{id}^*$ can be computed as follows: for given $(K, L, N)$, $P_1$ and $P_2$, denoting the probability that a tag collides during a frame (of size $L$) and that the tag collides through $K/L$ consecutive frames, respectively, are given by $P_1 = 1 - (1 - 1/L)^{N-1}$, $P_2 = 1 - (1 - (1/L)^{K-1})^{N/L}$, from which we obtain

$$P_{id}^* = 1 - \left(1 - \left(1 - \frac{1}{L}ight)^{N-1}\right)^{K/L}$$

(2)

We need to find the value of $L$ in $A_K$ that maximises $P_{id}^*$. Temporarily, we remove any restrictions on $L$, such that $L \in \mathbb{A}_K$ or $L$ should be an integer, and seek the value of $L$ ($L > 0$) by maximising $P_{id}^*$, say $L^\star$. By setting $e = KN (c \geq 1$ for $K \geq N)$ and $L = KN (0 < \kappa \leq c)$ in (2), we can express $P_{id}^*$ as a function of $\lambda$, i.e.

$$P_{id}^* = 1 - \left(1 - \left(1 - \frac{1}{L^\star}\right)^{N-1}\right)^{K/L}$$

(3)

Fixed $K$ and $N$, we determine the value of $\lambda$ maximising $P_{id}^*$, say $\lambda^\star$ (thus, $L^\star = L^\star \times N$). It is noteworthy that $L^\star$ does not depend on $K$ because it suffices to minimise $(1 - (1 - 1/(L^\star N))^{1/4})$ in (3). We now consider the asymptotic regime of $P_{id}^*$, i.e. when $N$ is sufficiently large. For large $N$, $P_{id}^*$ can be approximated by $1 - (1 - e^{-\lambda g^\prime}(x))$, i.e.

$$P_{id}^* \rightarrow 1 - (1 - e^{-\lambda g^\prime}(x))$$ as $N \rightarrow \infty$

For convenience, we set $x = 1/\lambda$ (thus, $1/\lambda \leq x$) and define the mapping $f(0, \infty) \rightarrow \mathbb{R}$ by

$$f(x) = 1 - e^{-\lambda g^\prime}(x)$$

Theorem 1: The mapping $f(0, \infty) \rightarrow \mathbb{R}$ has the unique minimum at $x = 2$.

Proof: Taking the natural logarithm of $f(x)$, we have

$$\ln f(x) = cx \ln (1 - e^{-c}) = -c \ln (e^c) \ln (1 - e^{-c})$$

Consider the function $g(y) = \ln (1 - y)/(1 - y)$, $0 < y < 1$. The second derivative of $g(y)$ is given by

$$g^\prime(y) = \frac{\ln (1 - y) - 1}{y^2} + \frac{\ln y}{y - y^2} + \frac{2}{y - 1}$$

Using the inequality $\ln (1 - y) \geq -1 - y - 1/(2y^2)$, we obtain $g^\prime(y) \leq 0$ (i.e. the function $g(y)$ is concave). On noting that $g(y) = g(1 - y)$, it follows by the concavity of $g(y)$ that, for all $y \in (0, 1)$,

$$g(y) = \frac{g(y) + g(1 - y)}{2} \leq \frac{g(0) + g(1 - 0)}{2} = g\left(\frac{1}{2}\right)$$

From the equation $y = e^{-c} = 1/2$, we can conclude that the function $f(x)$ attains its minimum at $x = 2$. The minimum is unique because $g(y)$ is strictly concave on $(0, 1)$.

From Theorem 1, we can easily derive the following fact.

Corollary 1: Asymptotically, we have

$$L^\star_N \approx 1/\ln 2$$ and $L^\star_N \approx N/\ln 2$

Corollary 2: For given $(N, K)$, the maximum achievable probability of individual successful transmission is asymptotically given by

$$P_{id}^* \approx 1 - 2^{-\gamma}$$

with $\gamma = K/\ln 2$.

Corollary 2 implies that in optimally configured ID processes (i.e. where the frame sizes are set according to the formula $L_{opt} = N/\ln 2$), the probability that a tag fails to be identified (i.e. $P_{id}^* \approx 1 - P_{id}^\prime$) approaches zero exponentially fast as a function of the number of frames. More specifically, it is reduced by half for every reading frame.

We observe in relation to the solution to the OP that, for sufficiently large $N$,

$$L_{opt} \approx \arg\min_{L \in \mathbb{A}_K} \{L - N/\ln 2\}$$

Conclusion: In this Letter, we considered an OP related to the selection of appropriate FSA frame sizes under a fixed ID time constraint. The problem is to determine the optimal frame size in the context of maximising the probability of successful ID. Concerning the OP, we considered the case of large $N$ (number of tags) and showed the existence of a unique solution, thereby obtaining the result that $L_{opt} = \arg\min_{L \in \mathbb{A}_K} \{L - N/\ln 2\}$.

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